Seat No. ________________

# HQ-003-1012008

## B. Sc. (Sem. II) (CBCS) (W.E.F. 2016) Examination

**April - 2023**

## Mathematics : MATH-02(A)

***(Geometry, Calculus and Matrix Algebra)***

***(Old Course) (Theory)***

**Faculty Code : 003**
**Subject Code : 1012008**

Time : $2\frac{1}{2}$ / Total Marks : **70**

**Instructions :** (1) All questions are compulsory.
(2) Figures written to the right side indicates full marks of the question.

**1** (a) Answer the following questions briefly : **4**

(1) Define Sphere.

(2) Find the Sphere for which (1, 1, 0) and (0, 1, 1) are the extremities of a diameter.

(3) Define Cylinder.

(4) Write the equation of cylinder with generator is parallel to X - axis and enveloping curve is $x^2 + y^2 + z^2 = a^2$.

(b) Answer any one briefly : **2**

(1) Find centre and radius of sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z - 11 = 0$.

(2) Find equation of right circular cylinder with radius 2 and axis $\frac{x-1}{2} = y - 2 = \frac{z-3}{2}$.

(c) Answer any one briefly : **3**

(1) Prove that the plane $lx + my + nz = p$ will touch the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$ if $(lu + mv + nw + p)^2 = (l^2 + m^2 + n^2)(u^2 + v^2 + w^2 - d)$.

(2) Find equation of cylinder whose generator is parallel to $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ and enveloping the curve is $x^2 + y^2 + z^2 = a^2$.

(d) Answer any one briefly : **5**

(1) Derive the equation of cylinder of which generator remain parallel to the line $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ and passing through a guiding curve $ax^2+2hxy+by^2+2gx+2fy+c=0,\ z=0$.

(2) Prove that the plane $x+2y-z=4$ cuts the sphere $x^2+y^2+z^2-x+z-2=0$ in a circle of radius unity and find the equation of the sphere which has this circle for one of the great circle.

**2** (a) Answer the following questions briefly : **4**

(1) Define Isolated Point.

(2) Define Connected Set.

(3) Define Partial Differentiation.

(4) State Schwartz's Theorem.

(b) Answer any one briefly : **2**

(1) Evaluate $\lim\limits_{(x,y)\to(0,0)} \frac{x-y}{x+y}$

(2) If $x^3+y^3+z^3=3xyz$ then find $\frac{\partial z}{\partial x}$.

(c) Answer any one briefly : **3**

(1) Discuss the existence of $\lim\limits_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ by deriving iterative limits.

(2) Find the first order partial derivative of

$f(x, y)=x\tan\frac{y}{x}$ at $(4, \pi)$.

(d) Answer any one briefly : **5**

(1) If $f(x,y)=\begin{cases}\frac{x-y}{x+y} & \text{if } (x, y)\neq(0, 0)\\ 1 & \text{if } (x, y)=(0, 0)\end{cases}$ then prove that $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ does not exist.

(2) State and Prove Euler's Theorem for homogeneous function of two variables.

**3** (a) Answer the following questions briefly : **4**

(1) Define local mixima.

(2) Define global mixima.

(3) Define Extreme point.

(4) Define Jacobian.

(b) Answer any one briefly : **2**

(1) If $x = r\cos\theta$ and $y = r\sin\theta$ then prove that $\frac{\partial(x, y)}{\partial(r, \theta)} = r.$

(2) State Maclaurian's expansion for function of several variables.

(c) Answer any one briefly : **3**

(1) If $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ then find approximate value of $f(1.9, 2.01, 4.8)$.

(2) If $x = \cos\theta,\ y = \sin\theta$ and $z = z$ then prove that $J\left(\frac{x, y, z}{r, \theta, z}\right) = 1.$

(d) Answer any one briefly : **5**

(1) State and Prove Taylor's expansion for function of several variables.

(2) If $\frac{4}{x} + \frac{9}{y} + \frac{16}{z} = 25$ then by Lagrange's method obtain the value of $x$, $y$ and $z$ which make $x + y + z$ is minimum.

**4** (a) Answer the following questions briefly : **4**

(1) Define Singular Matrix.

(2) Define Transpose of Matrix.

(3) Define Rectangular Matrix.

(4) Define Rank of Matrix.

(b) Answer any one briefly : **2**

(1) Prove that $M = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$ is nilpotent of index 2.

(2) Define Upper Triagular and Lower Triangular Matrix.

(c) Answer any one briefly : **3**

(1) Prove that matrix $A = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ has period 2.

(2) If $M = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 7 \\ 3 & 4 & 4 \end{bmatrix}$ then find $M^{-1}$.

(d) Answer any one briefly : **5**

(1) Prove that every square matrix can be expressed uniquely as the sum of a symmetric and skew – symmetric matrix.

(2) Find two non-singular matrices $X$ and $Y$ such that $XMY$ is in the normal form where $M = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$ and also find the rank of matrix $M$.

**5** (a) Answer the following questions briefly : **4**

(1) Define Characteristic Equation.

(2) Define Eigen Vectors.

(3) Find the characteristic polynomial of matrix $R = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

(4) Define Homogeneous system of linear equation.

(b) Answer any one briefly : **2**

(1) Verify Cayley – Hamilton theorem for matrix $M = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$.

(2) Prove that Eigen values of Hermitian matrix is real numbers.

(c) Answer any one briefly : **3**

(1) Find Eigen values and Eigen vectors of matrix $M = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$.

(2) Solve $x + y + z = 6,\ x - y + z = 2$ and $2x + y - z = 1$.

(d) Answer any one briefly : **5**

(1) State and Prove Cayley – Hamilton Theorem.

(2) The necessary and sufficient condition that the system of equation $AX = B$ is consistent is that the matrices $A$ and $[A: B]$ are of the same rank.

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